CS 583 – Computational Audio -- Fall, 2021

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Lecture 2

Additive/Fourier Synthesis; time domain vs frequency

Basics of musical signals: Fundamental frequency, harmonic series, partials, timbre.

Exploring resonance properties in musical instruments.

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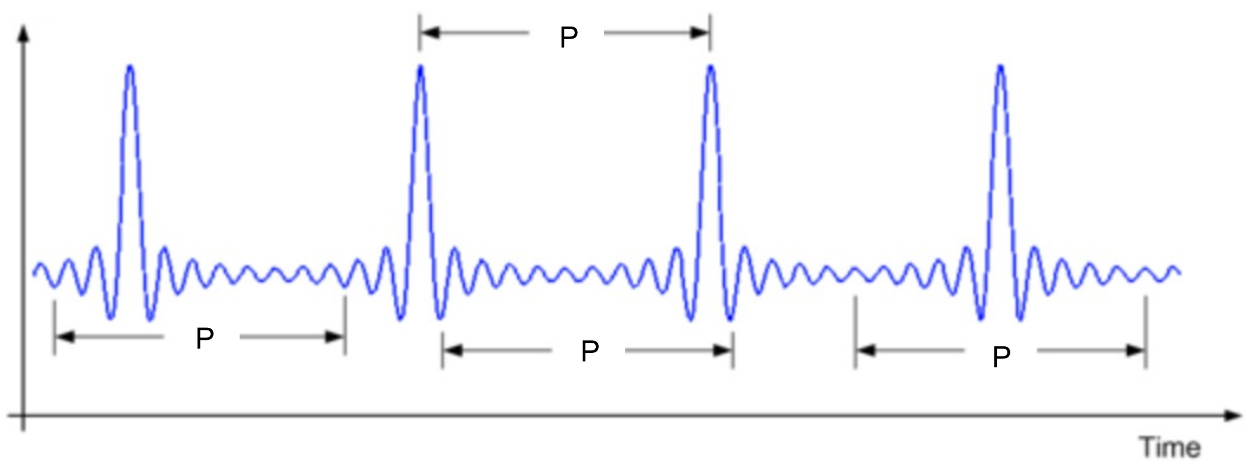
Jean-Bastiste Fourier

Jean-Baptiste Joseph Fourier (1768 – 1830) was a French mathematician and scientist who studied (among other things-- for example he was the first to explain the Greenhouse Effect) heat transfer in metals. In this investigation he realized the following remarkable fact:

Any periodic function with period P can be constructed by adding together a sequence (possibly infinite) of simple sine waves of various amplitudes and phases, with frequencies that are integer multiples of the fundamental frequency *f* = 1/P.

A periodic function is one which repeats its values at a period P:



Fourier/Additive Synthesis

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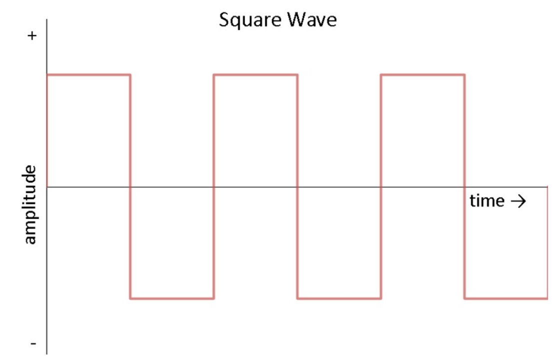
Example of Fourier/Additive Synthesis:



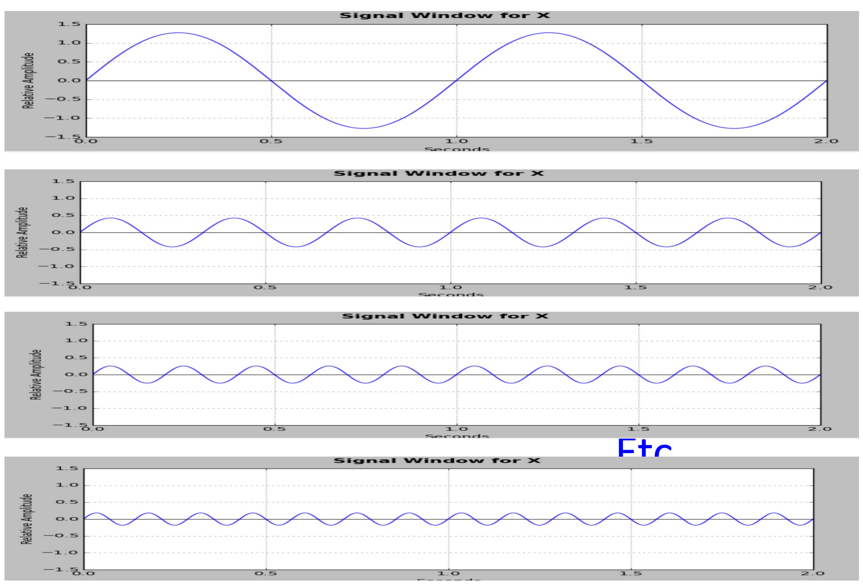
A square wave of frequency *f* and amplitude 1.0 can be created from the following infinite *Fourier Series*:

$(4/\pi)\big(\sin(2\pi f t) + \sin(2\pi\, 3f t) + \sin(2\pi\, 5f t) + \ldots\big)$

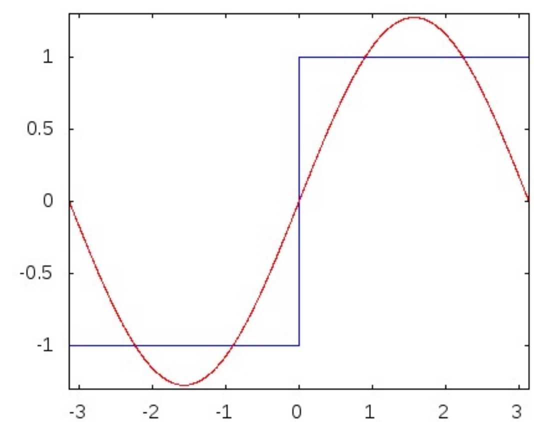
that is:

[(1,1.27,0), (3,0.42,0), (5,0.25,0), (7,0.18,0), (9,0.14,0), ….]

shown here graphically (duration of 2 sec):



Etc.

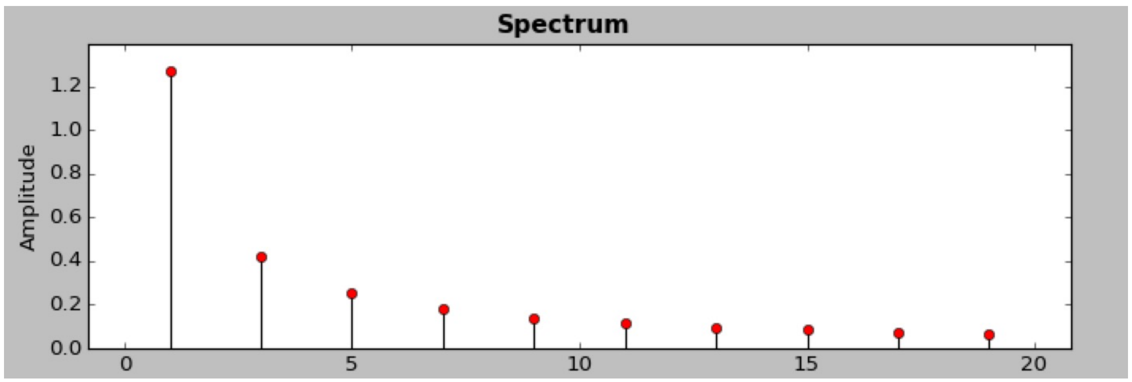


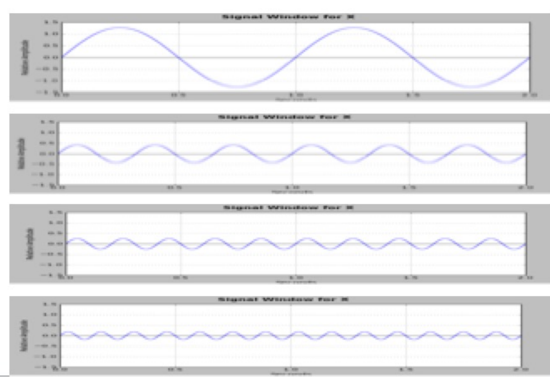
Fourier Analysis and Synthesis

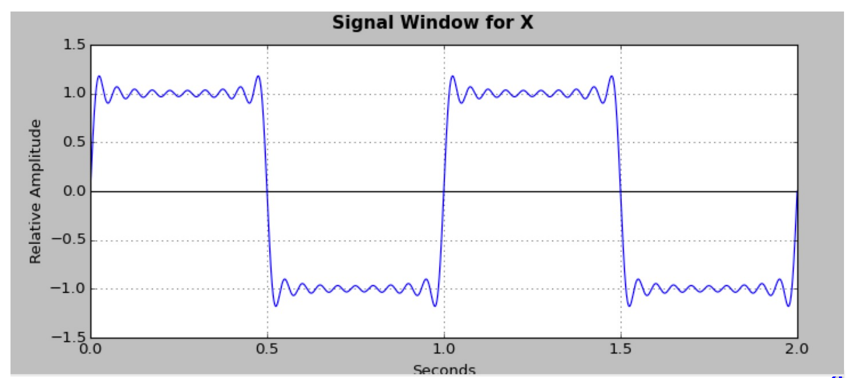
Spectrum: A graph of the amplitude and frequency of the components of a sinusoid is called a **spectrum**; here is a spectrum of a wave consisting of the first ten terms of the Fourier Series for a square wave.

[(1,1.27,0), (3,0.42,0), (5,0.25,0), (7,0.18,0), (9,0.14,0), (11,0.12,0), (13,0.1,0), (15,0.08,0), (17,0.07,0), (19,0.07,0)]

(amp, freq, phase)







Fourier Analysis and Synthesis

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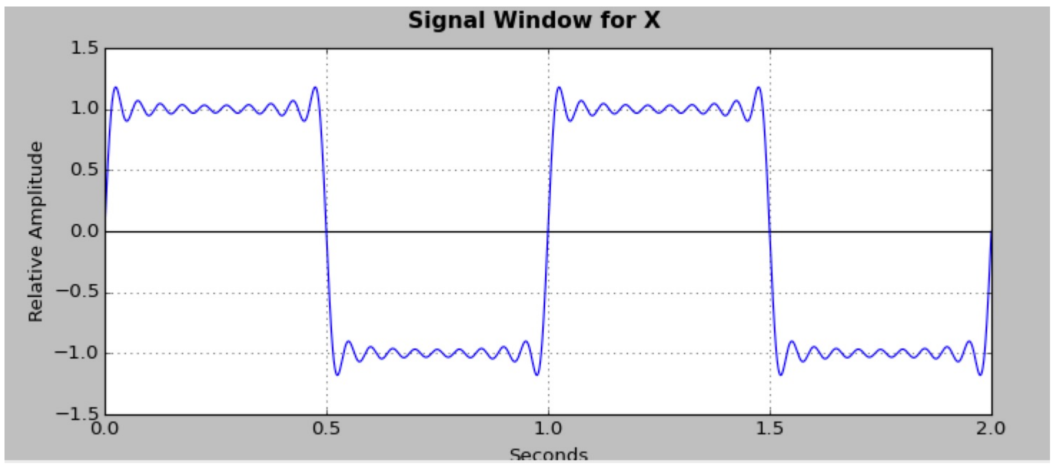
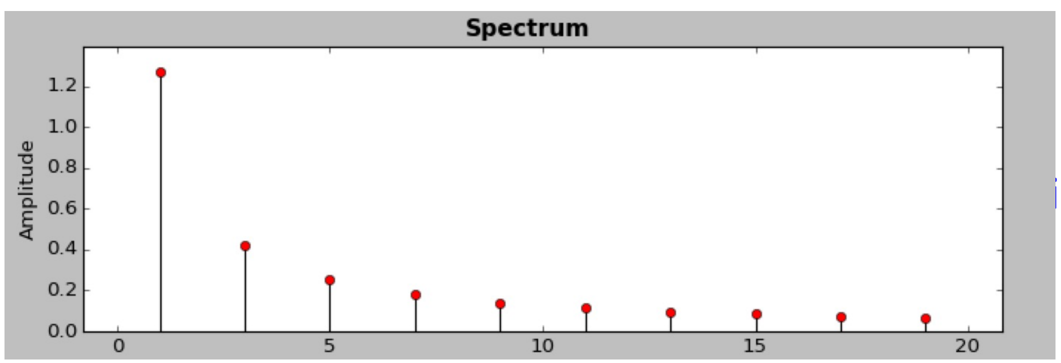
Fourier provided a way of looking at a signal in two equivalent ways, as a graph of amplitude of the signal vs time, the Time Domain, or as a graph of frequencies vs amplitudes (the Frequency Domain):

Fourier Analysis using FFT



Time Domain

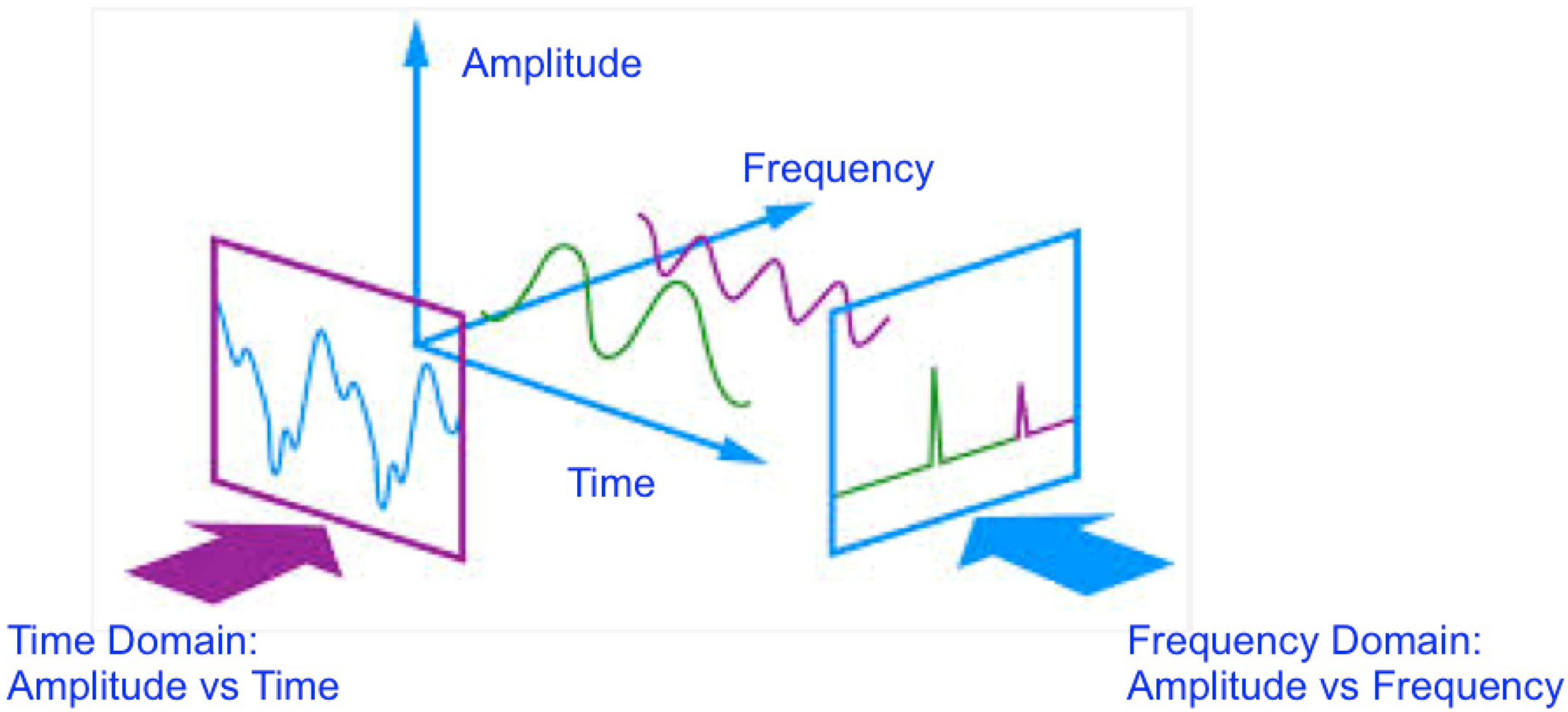
Fourier Synthesis using Inverse FFT



in

Fourier Analysis and Synthesis

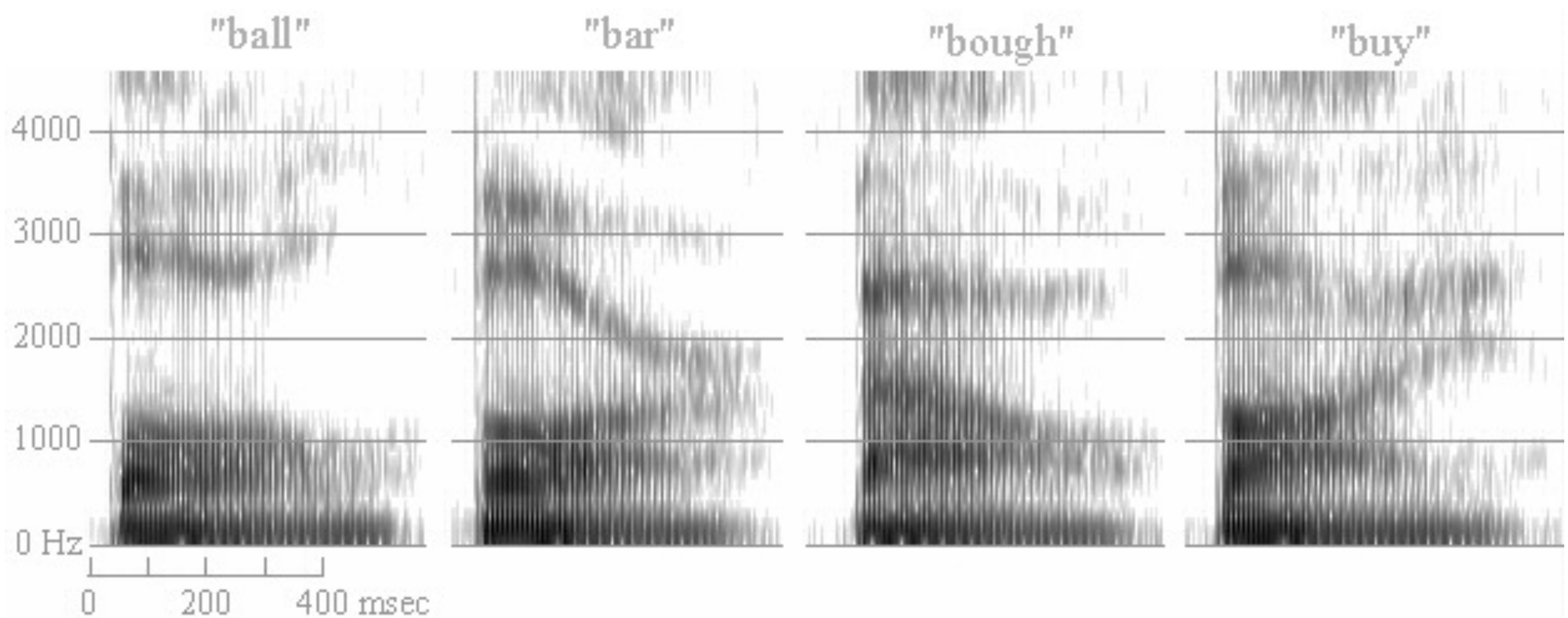
It can NOT be overemphasized how important these results are for understanding sound (which has three dimensions); these are two different, but completely equivalent ways of viewing the same phenomenon, and the pair of transforms are **lossless** and **very efficient**: O(N log N).



Fourier Analysis and Synthesis

Spectrograms: You can also add time to a spectrum by dividing a signal into small "windows" (perhaps 200 ms), calculating the spectrum for each, and showing how the spectrum changes over time. These are called spectrograms.

Typically these are displayed as frequency (y axis) vs time (x axis) with the amplitude of the frequency shown by greyscale:



Fourier Analysis and Synthesis

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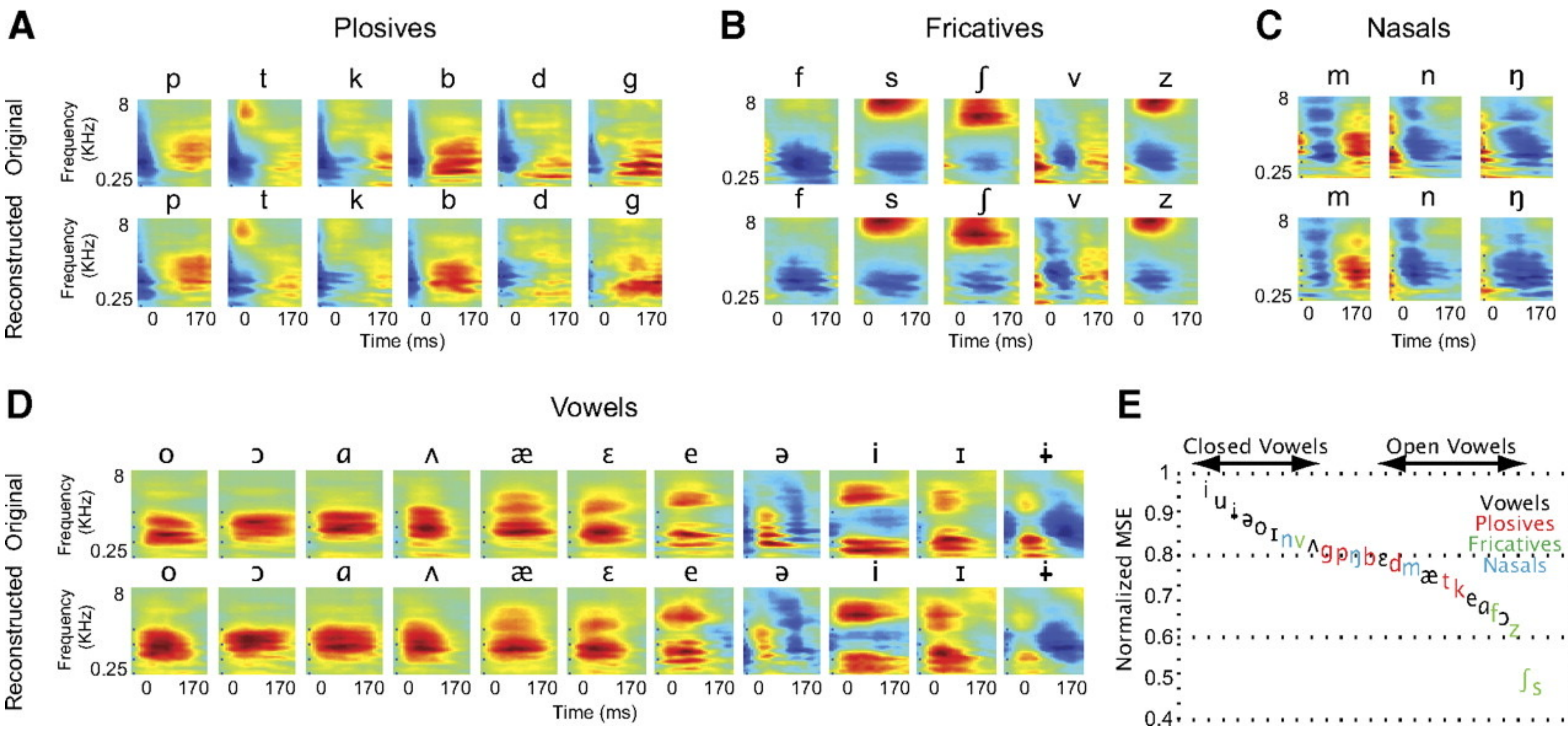
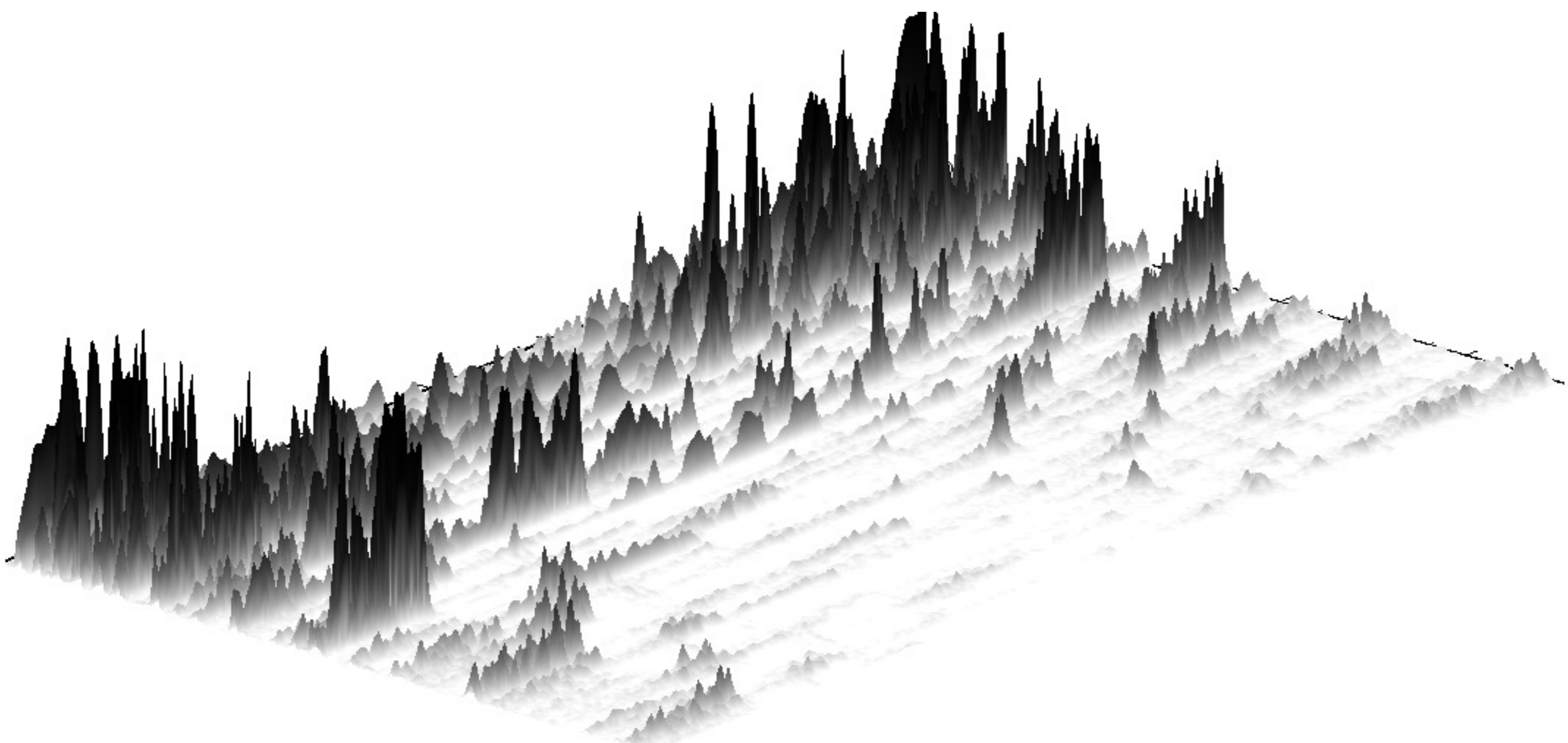
Or by color:



Fourier Analysis and Synthesis

Or in faux 3D:



Fourier Analysis and Synthesis

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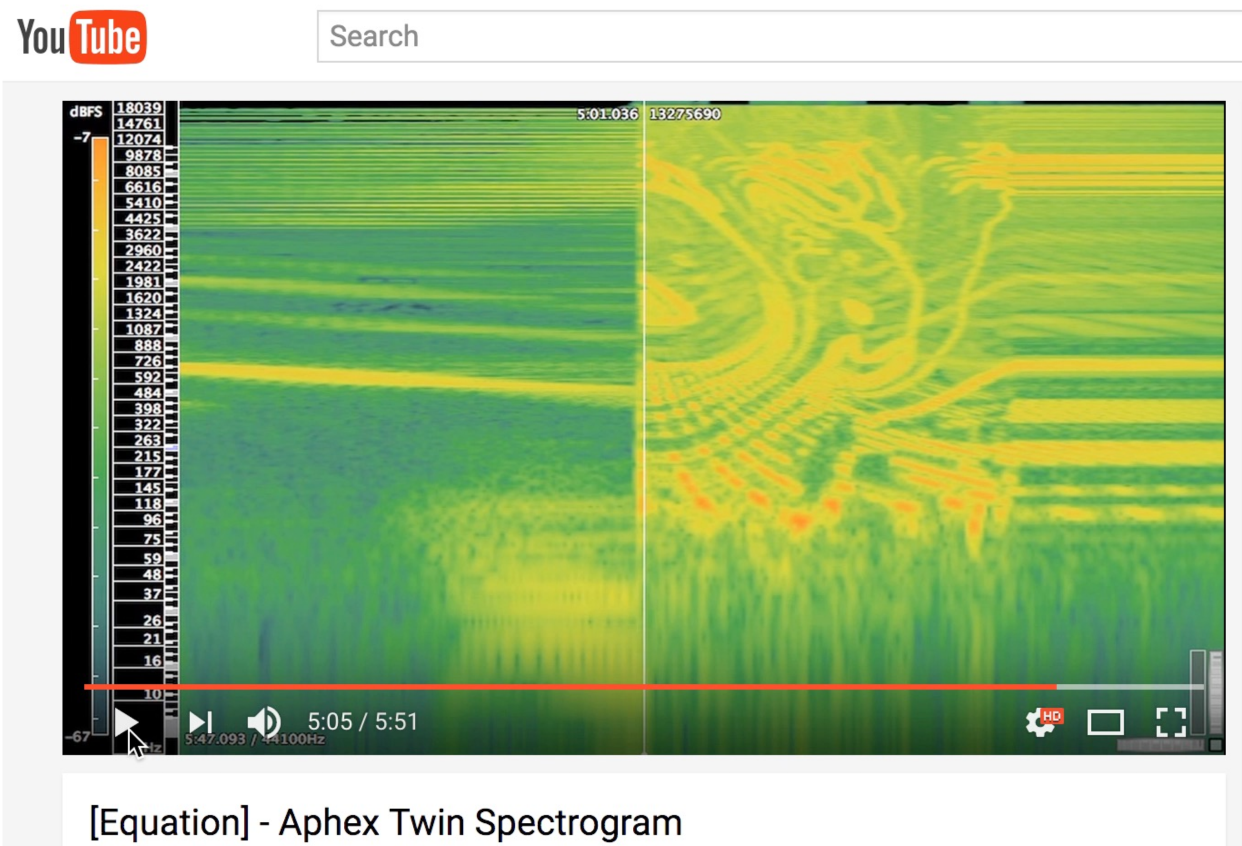
Digression for fun….. spectrogram artwork can be produced by various applications which take an image and create sounds that produce the given image…. . .



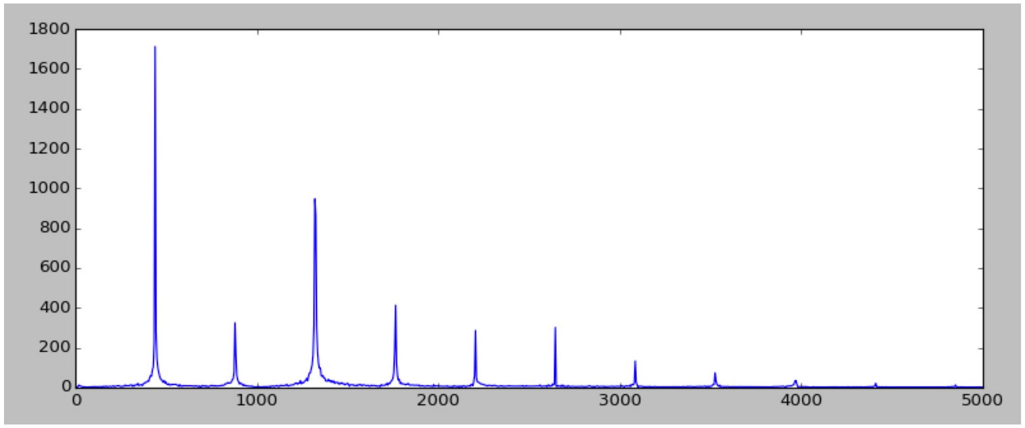
Harmonic Series

Harmonic Series: A series of integer multiples of a particular lowest, fundamental frequency is called a *Harmonic Series* and each component is called a Harmonic or Overtone:

Example (in Hz): 440, 880, 1320, 1760, 2200, 2640, 3080, 3520, 3960, ….

A periodic signal can always be constructed from such a sequence, although in additive synthesis of musical signals we will not restrict ourselves only to such sequences. Real musical signals always have more complex spectra.

Still, musical sounds are in large part based on such spectra; here is a clarinet playing A 440 Hz:



Harmonic Series

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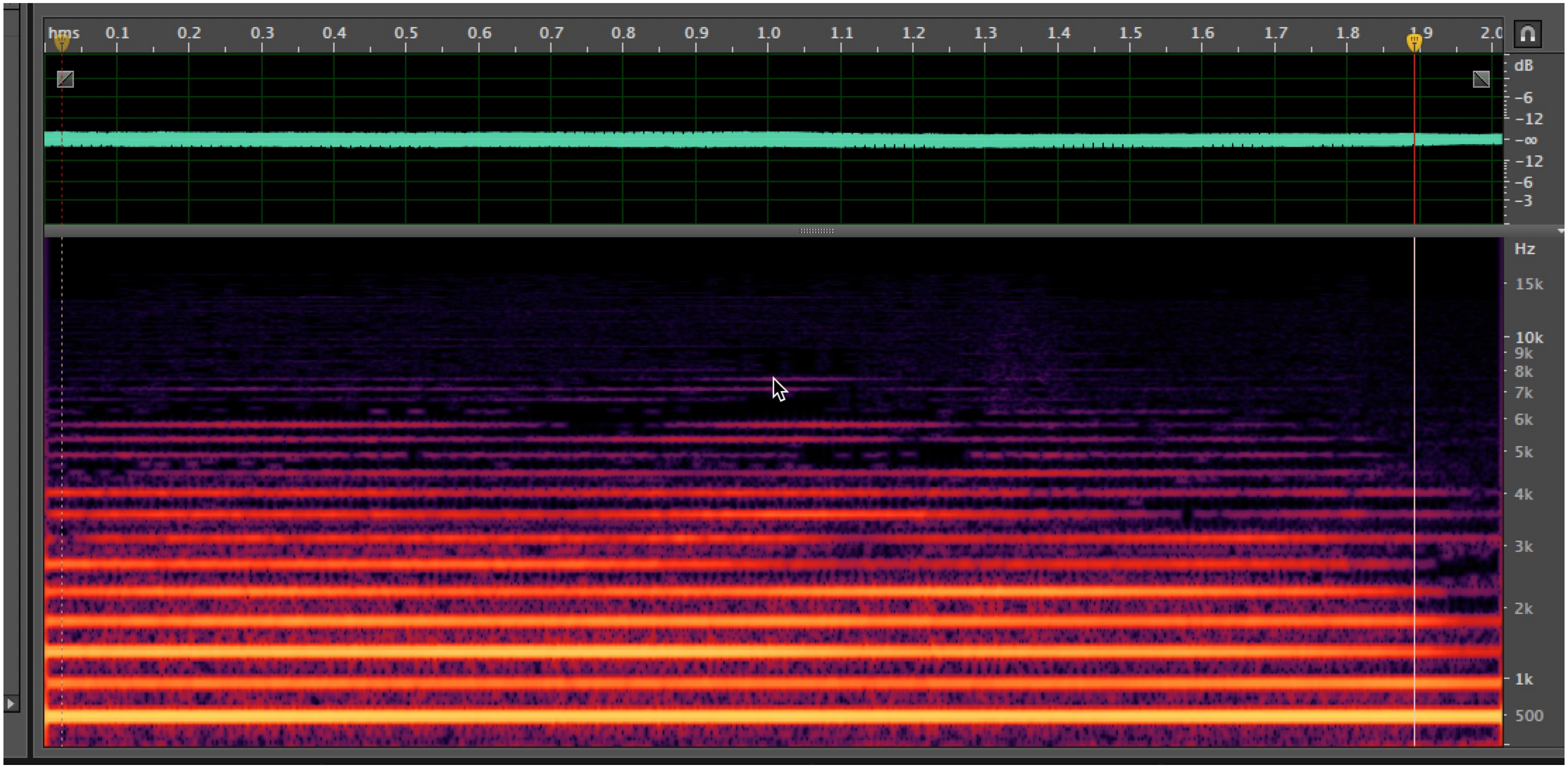
And here is a spectrogram of the same signal:



Musical Acoustics: String Vibration

Q: Why is the Harmonic Series so important?

A: It corresponds to the resonance properties of two very common ways of making musical sounds: Strings and Pipes.

Waves in Strings: To understand how resonance works to produce sound waves, we have to understand several physical properties.

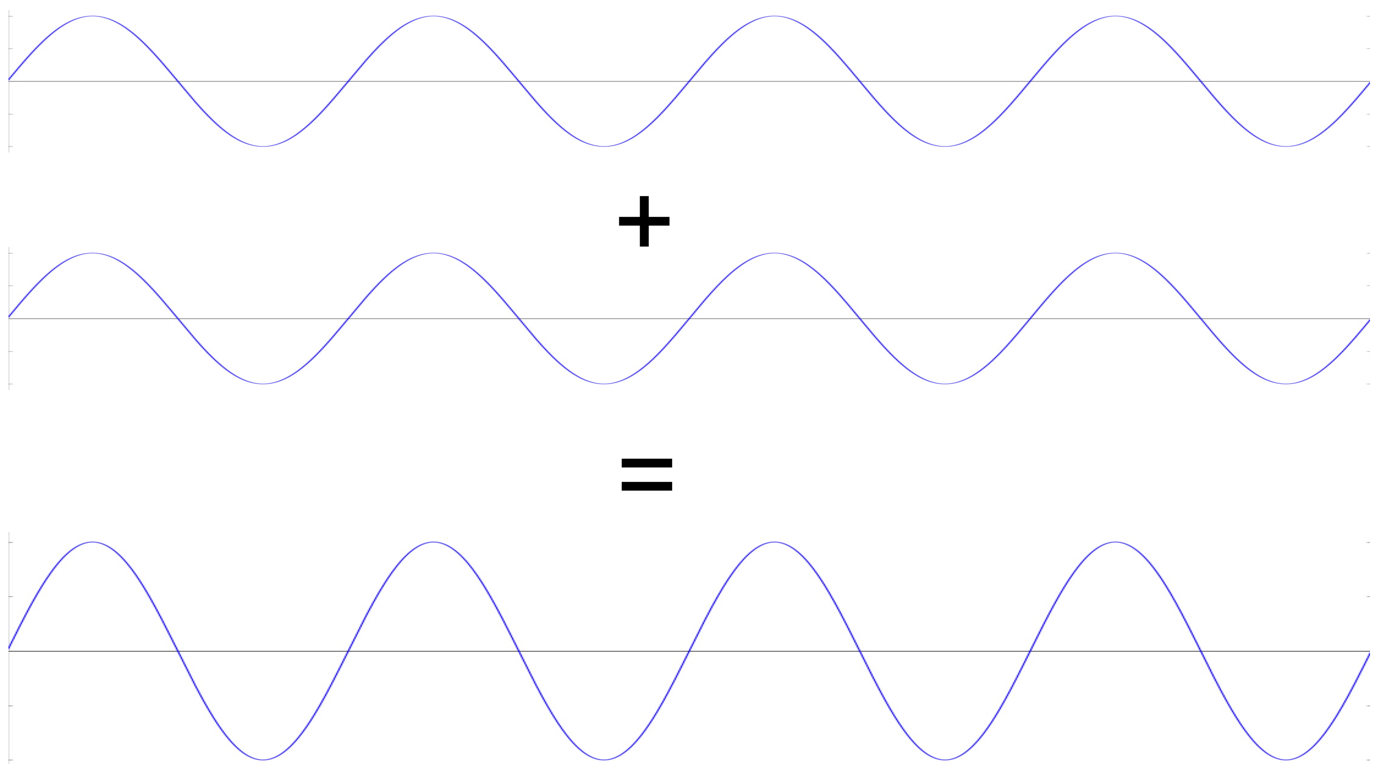
(1) We must understand how waves change when they bounce off a hard boundary vs a soft boundary:

Hard Boundary (amplitude inverts)   **Soft Boundary (not):**

Musical Acoustics: String Vibration

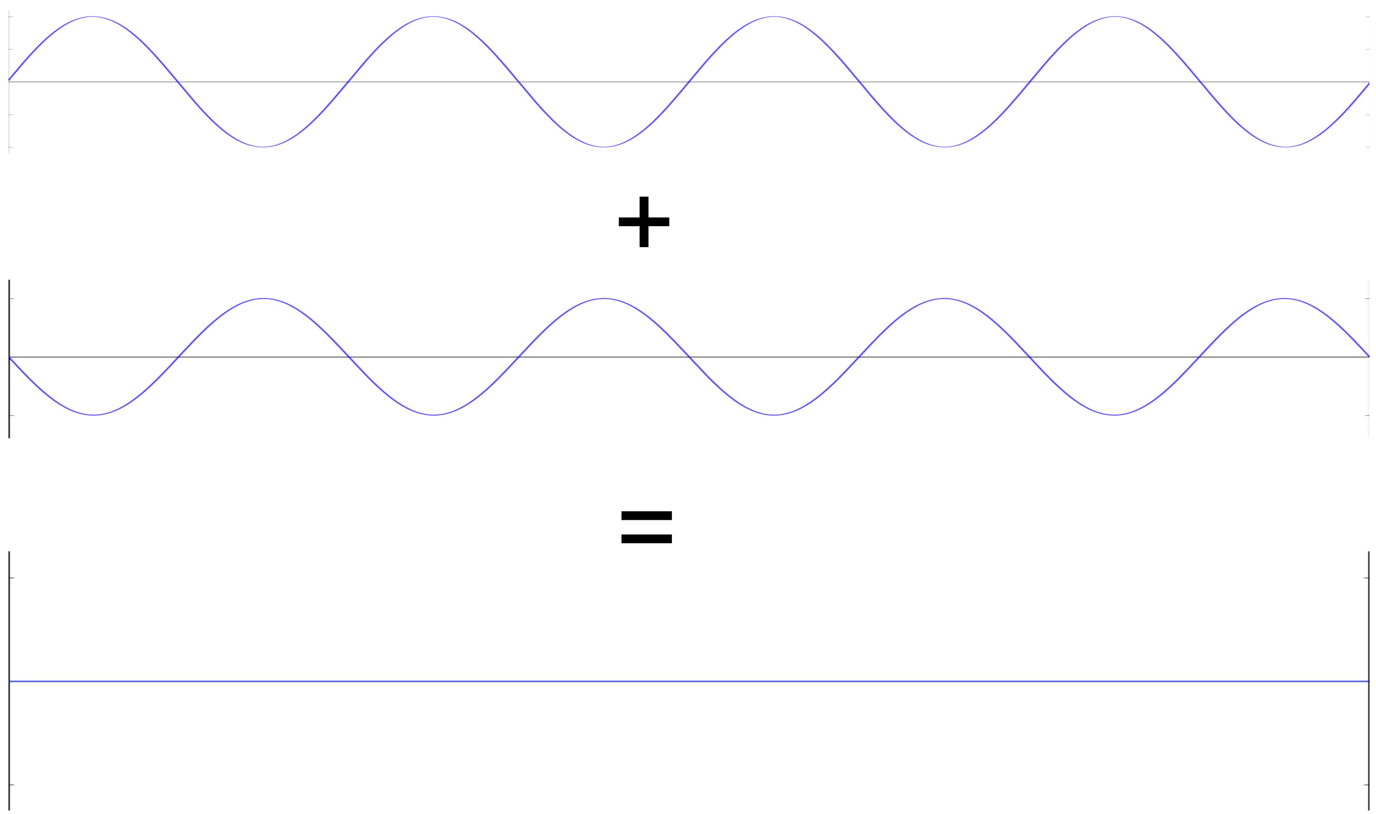
Thus, when a wave travels down a string, when it hits either fixed end, it reflects and inverts.

(2) We must understand how two waves of the same frequency can add together (constructive interference) to make a sum wave of larger amplitude and the same frequency:



Musical Acoustics: String Vibration

and how the same waves, if they are out of phase by π (= opposite amplitudes), can cancel out by destructive interference:



Musical Acoustics: String Vibration

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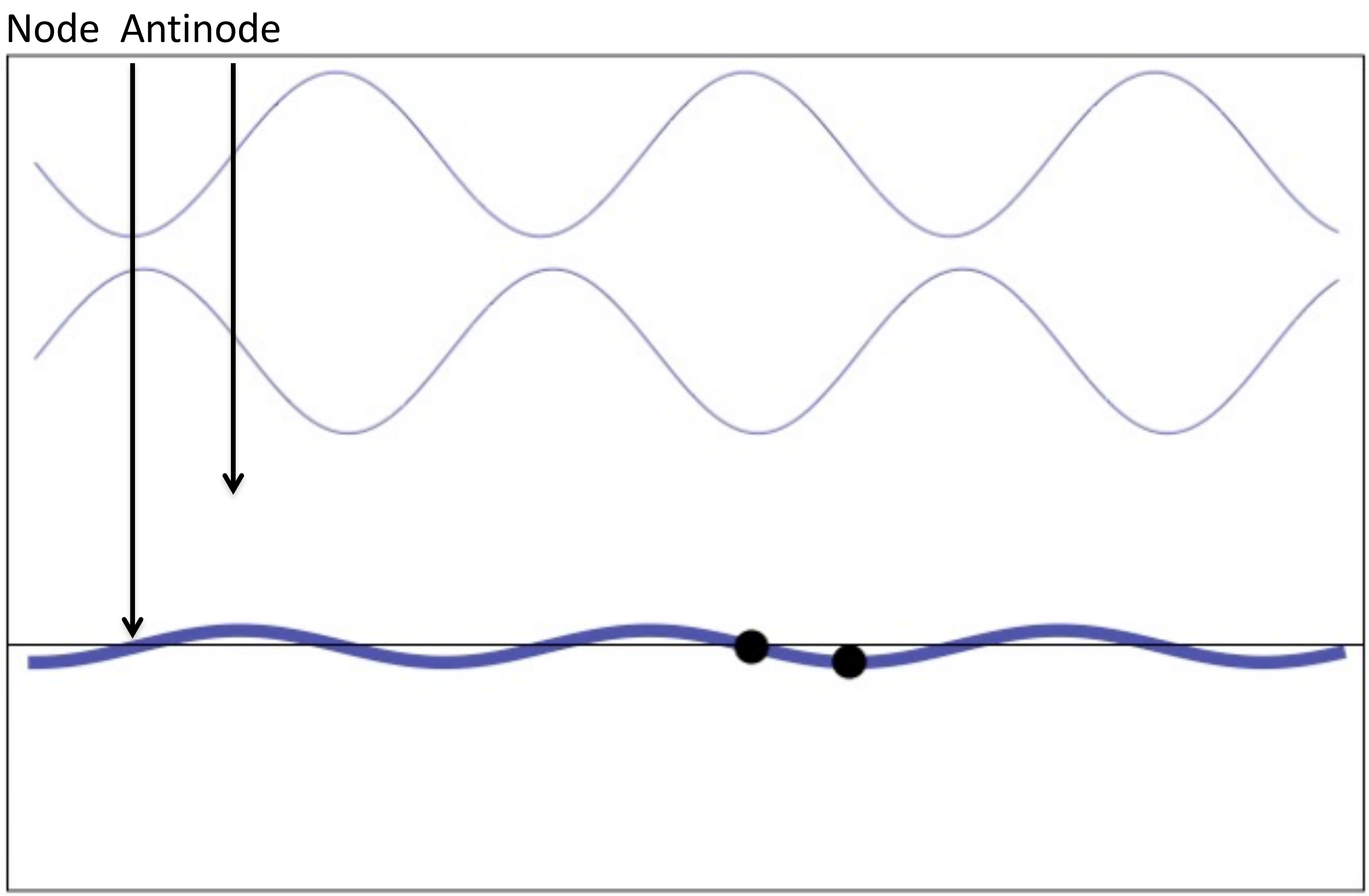
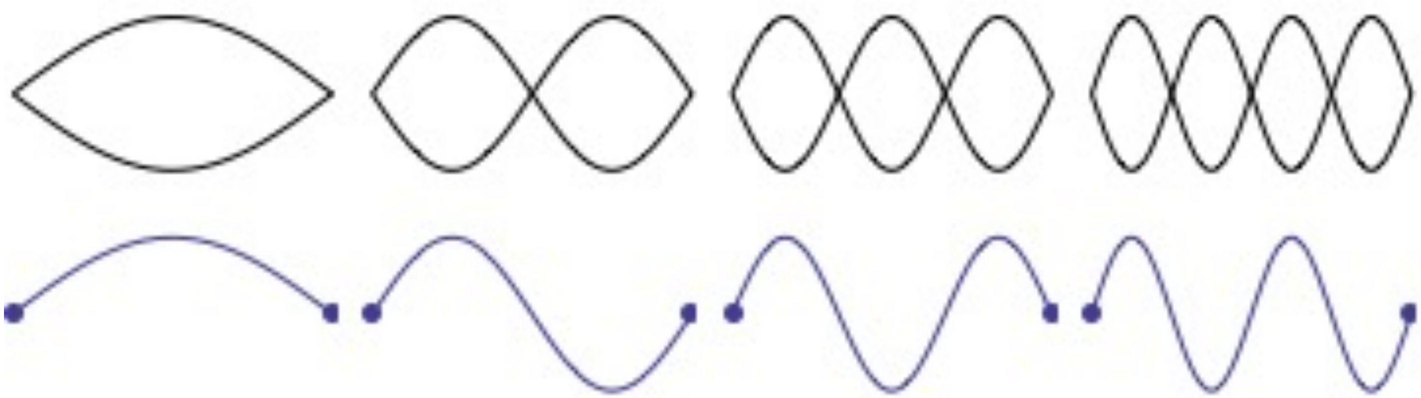
(3) Now, as a wave travels down and reflects back in the reverse direction, it will alternately move between constructive and destructive interferences, and produce a *standing wave*:



Musical Acoustics: String Vibration

Now one way such a wave can continue to reflect back and forth is if both ends of the string are at Nodes (which do not move). This is the basic property of Resonance, and produces to the Modes for a vibrating string:



Wavelengths:

$\frac{1}{2}\lambda$	λ	$3/2\ \lambda$	$2\ \lambda$

Frequencies:

f	$2f$	$3f$	$4f$

Musical Acoustics: String Vibration

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Thus, a guitar will emphasize the same Harmonic Series we have seen before:

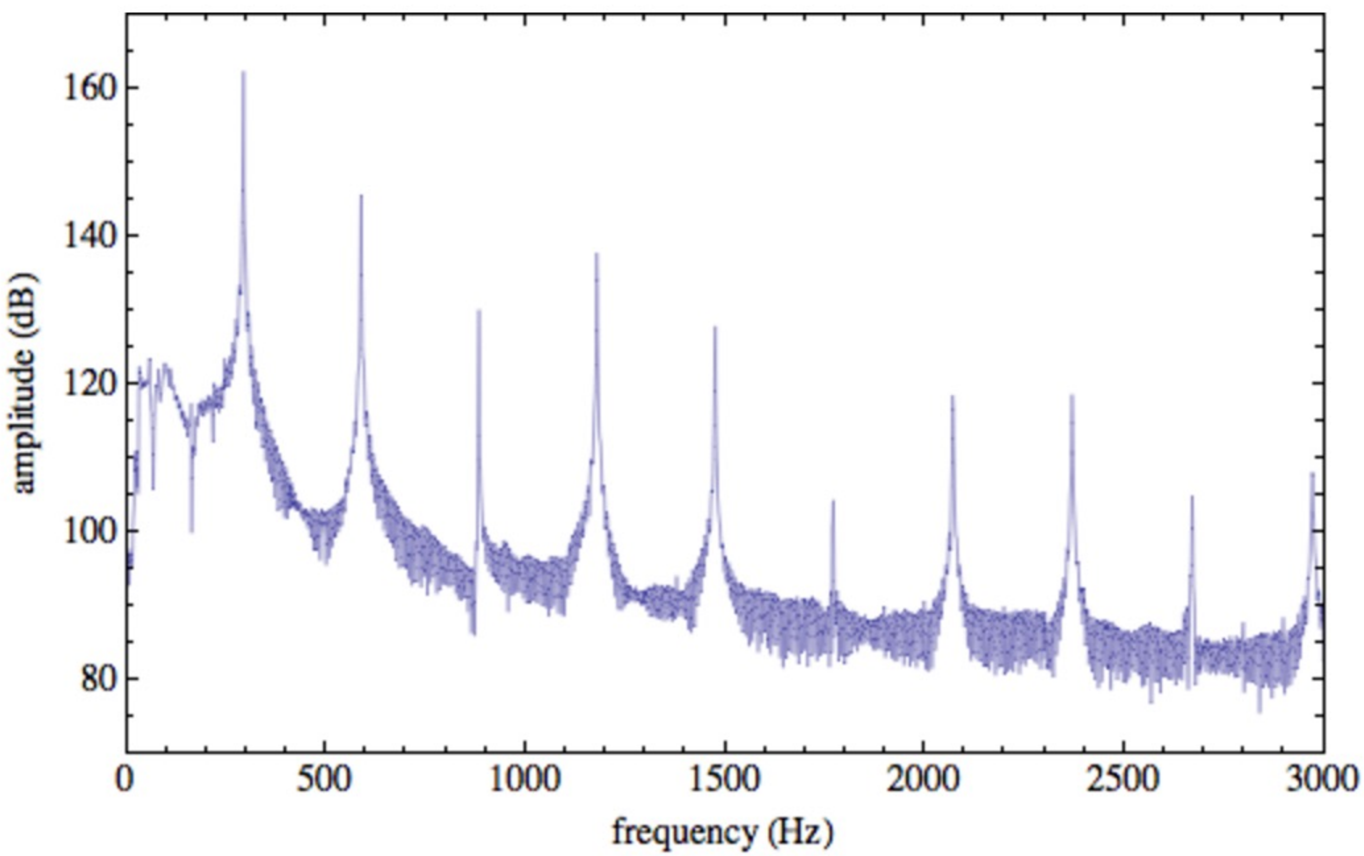
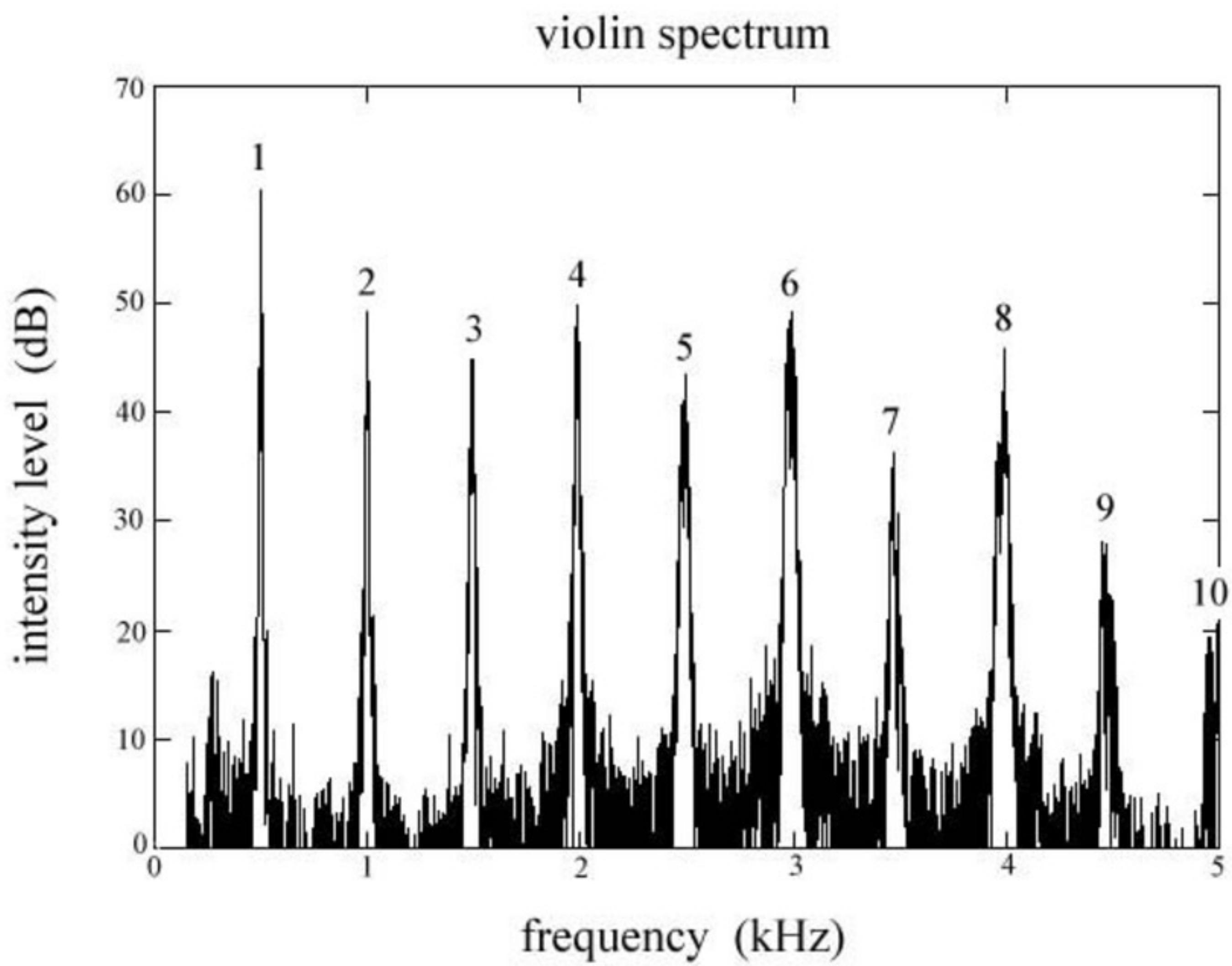


Figure 3: The frequency spectrum of note D4 played on guitar

D4 =
293.665 Hz

Musical Acoustics: Open and Closed Pipes



Musical Acoustics: Open and Closed Pipes

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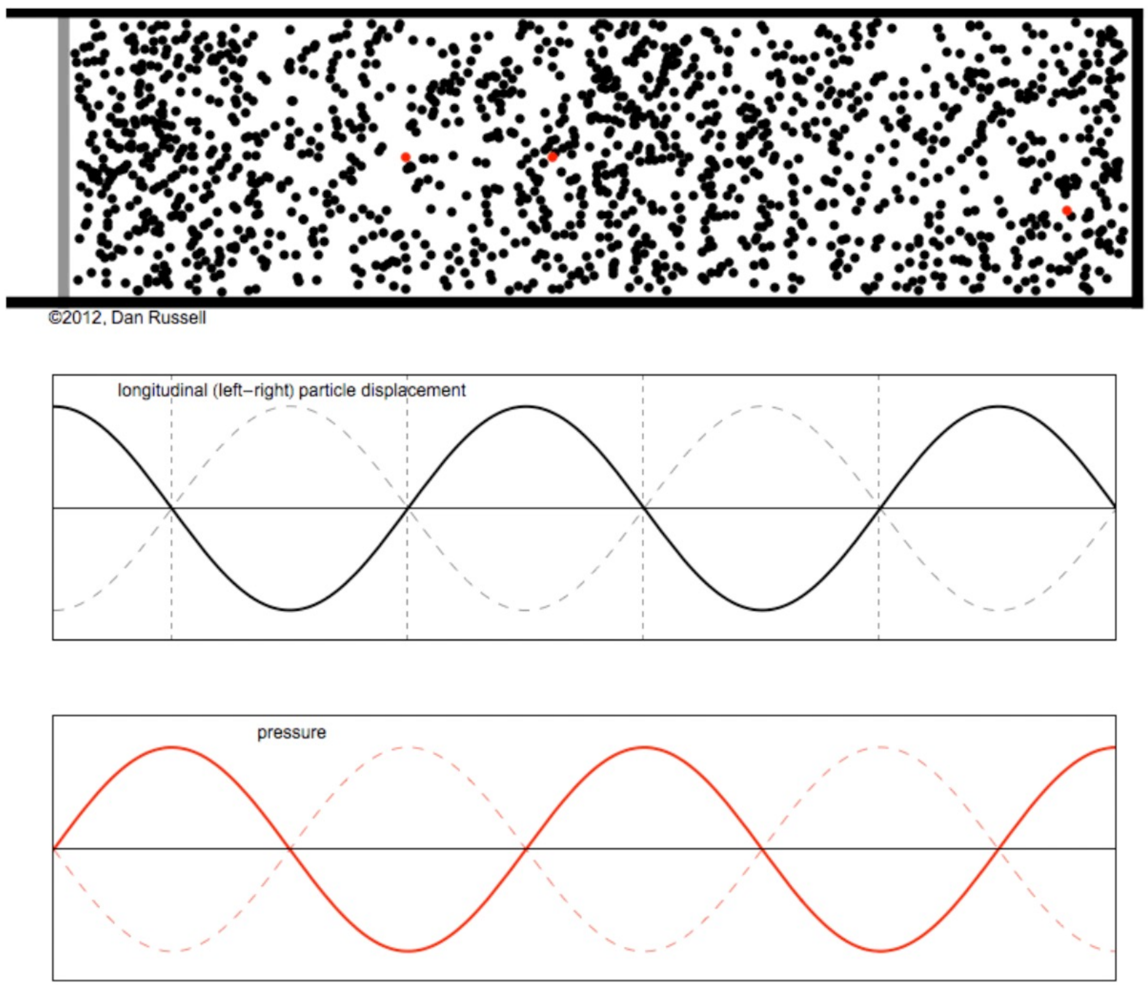
The same effect can be seen if we vibrate the air inside a pipe closed at both ends, but then the sound can not escape! So most wind instruments (clarinet, horns, saxiphones, flutes, etc.) are closed at one end and open at the other.

Resonance can also occur when a node occurs at the closed end and an antinode occurs at the closed end:



Musical Acoustics: Open and Closed Pipes

This means that the resonance modes in a half-closed pipe occur when there are

¼ , ¾ , 5/4 , 7/4, … etc.

wavelengths along the length of the pipe or frequencies

f, 3f, 5f, 7f, ….

(odd multiples of f_0).

Punchline: we would expect string instruments to have frequency components at all (or most) harmonics, and wind instruments to have frequency components at odd-numbered harmonics.

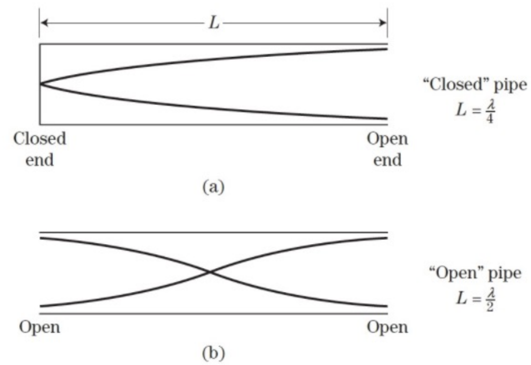
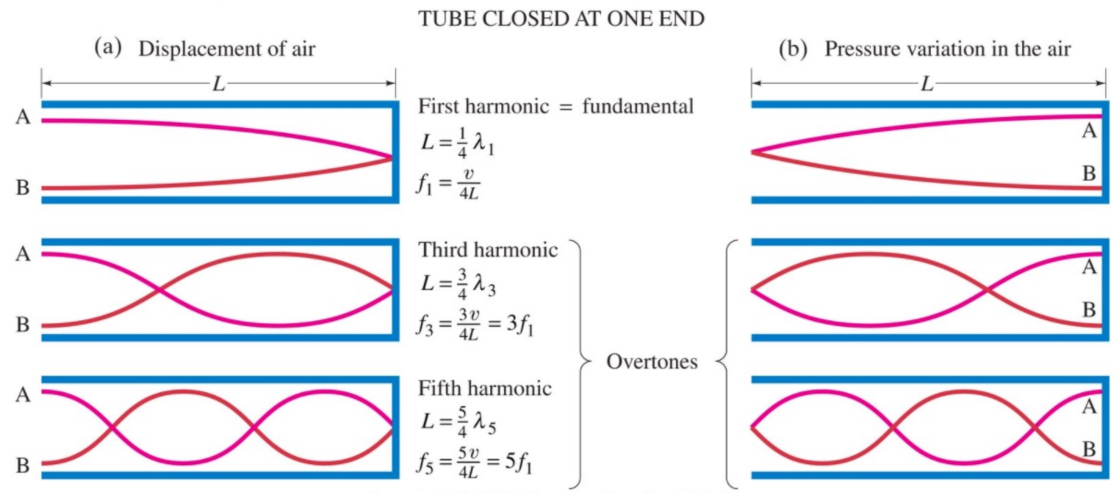


FIGURE 6–6

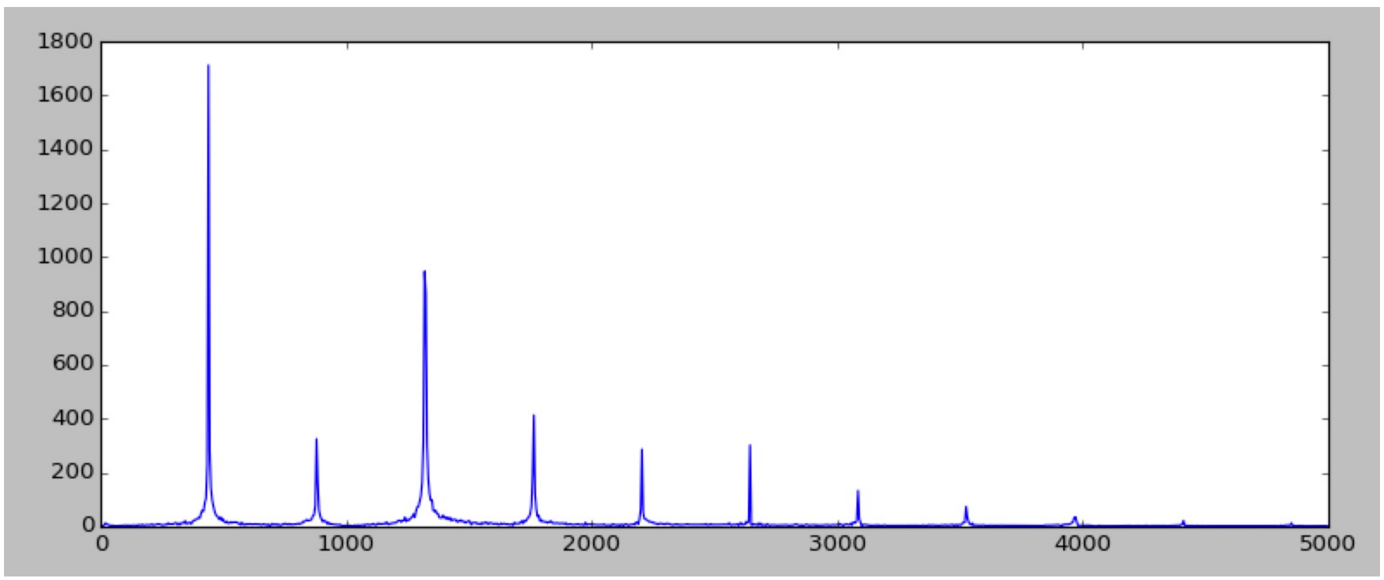
Standing waves in a pipe. (a) One end closed and (b) both ends open. Organ pipes are operated with this standing wave profile. The amplitude of the standing wave as the air blows is maximum and acts as an open end. Only the fundamental modes are shown.



Musical Acoustics: Open and Closed Pipes

In fact, this model is not completely accurate, and there are many other factors that come into play.

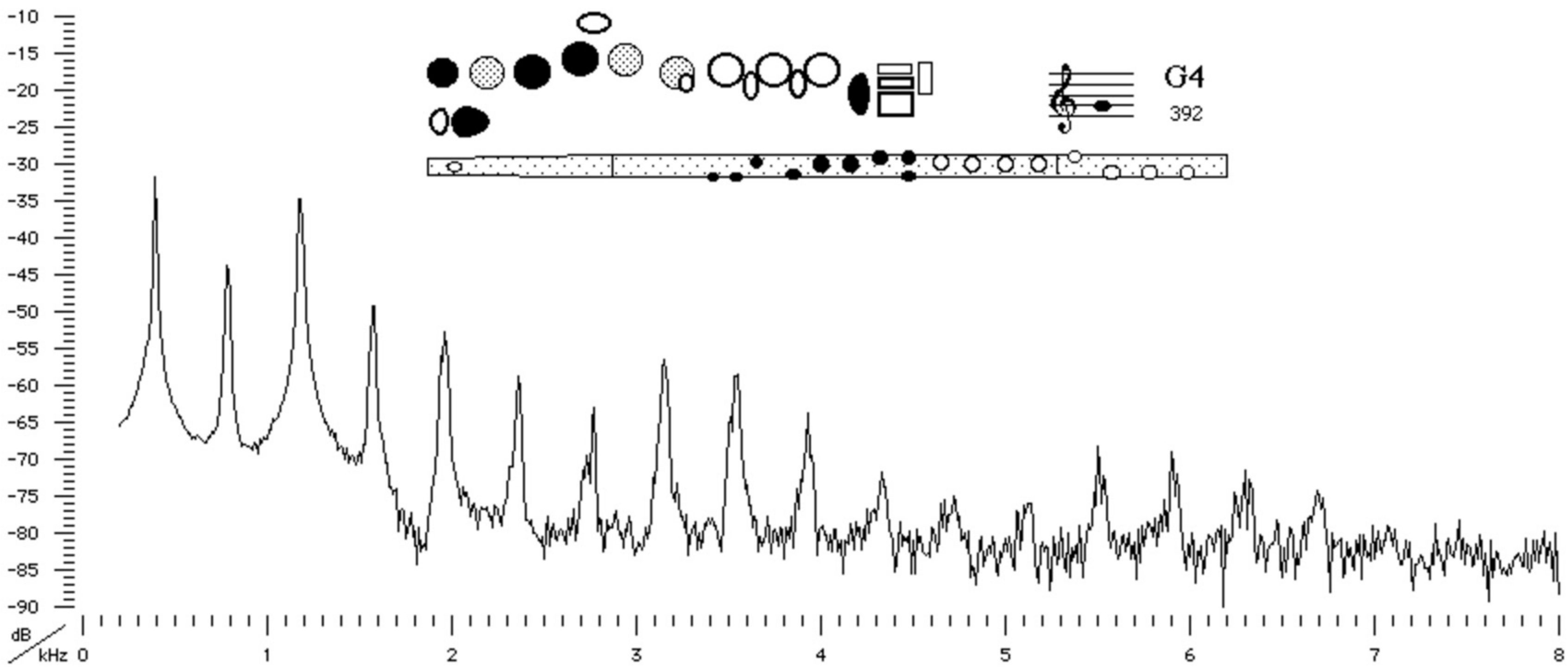
The clarinet spectrum does have strong first and third harmonics, but thereafter the odd harmonics are not so pronounced:



However, the first couple of harmonics have the most effect on the sound.

Musical Acoustics: Open and Closed Pipes

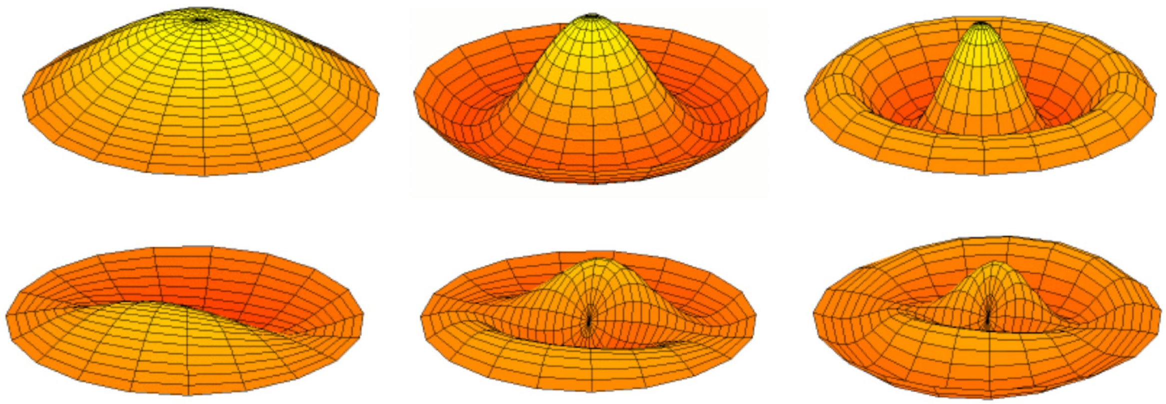
A similar situation exists with the flute:



Sound spectrum of a modern flute with a B foot played using fingering for G4.

Musical Acoustics: Vibrating Bodies

For many instruments, there are additional vibration modes induced by the body of the guitar, violin, drum, etc. that add additional resonances. These are harder to describe simply!



Drumhead Vibration Modes

Musical Acoustics: Vibrating Bodies

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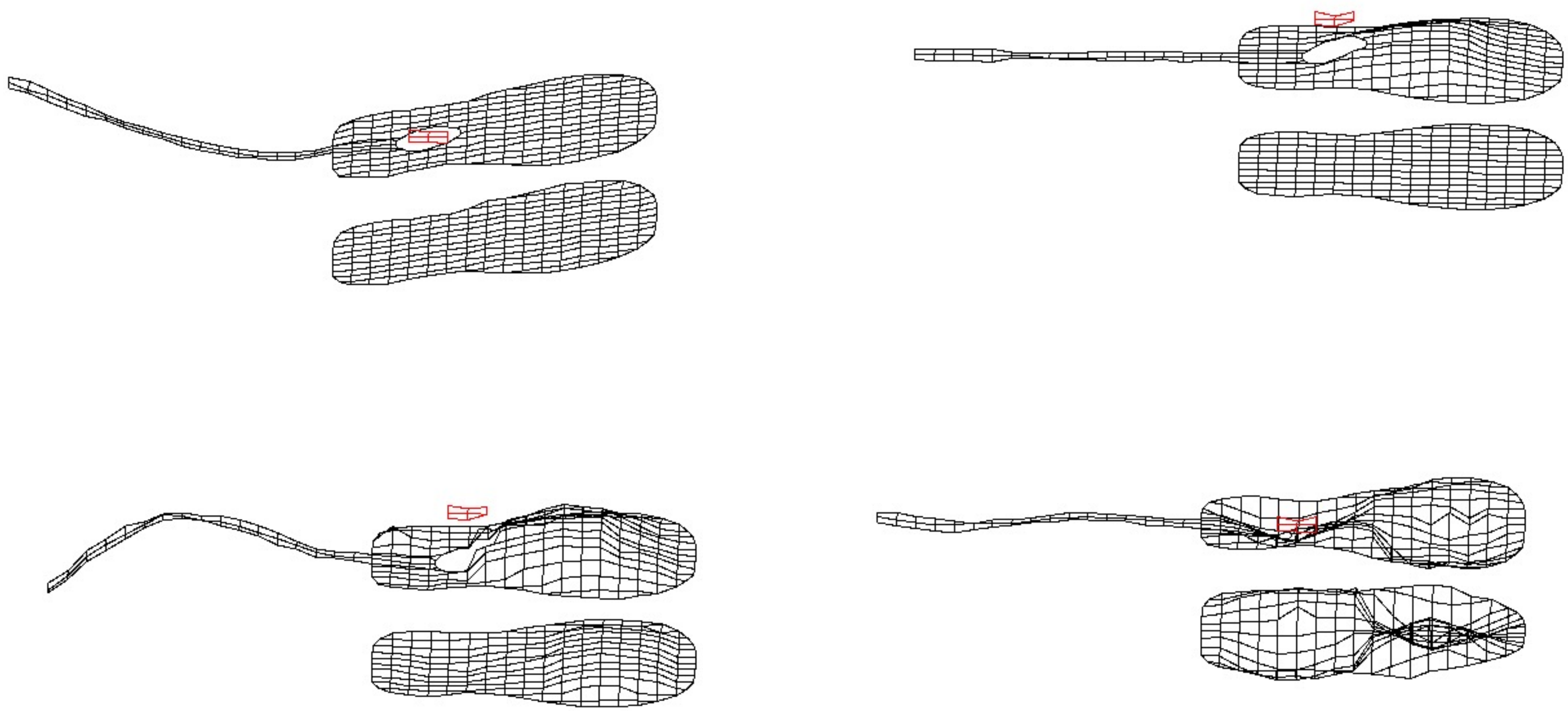
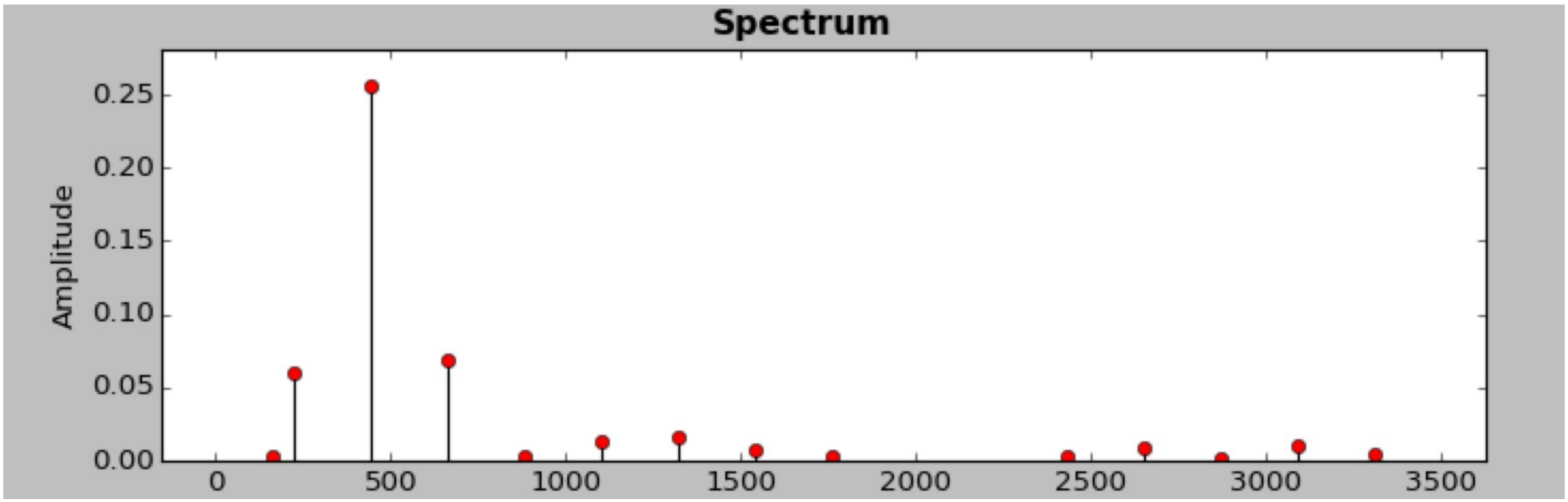


Musical Acoustics: Spectra

Here is another example (of a steel string guitar); in this one there are two component which are BELOW the fundamental, due to the resonance of the body of the guitar:



Musical Acoustics: Characteristics of Musical Signals

It is the characteristic spectra of musical instruments which is a major component of their timbre:

In music, **timbre** (/ˈtæmbər/ ***tam**-bər*, also known as **tone color** or **tone quality** from psychoacoustics) is the perceived sound quality of a musical note, sound, or tone that distinguishes different types of sound production, such as choir voices and musical instruments, such as string instruments, wind instruments, and percussion instruments, and which enables listeners to hear even different instruments from the same category as different (e.g. a viola and a violin)

--Wikipedia article on Timbre

The other important characteristics of a musical signal are:

- Pitch
- Loudness (amplitude)

And all of these can vary over time!

Musical Acoustics: Vocal Tract

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Even if you do not play a musical instrument, you use spectra all the time to communicate! The reason you recognize different vowel sounds is that they have different spectra:

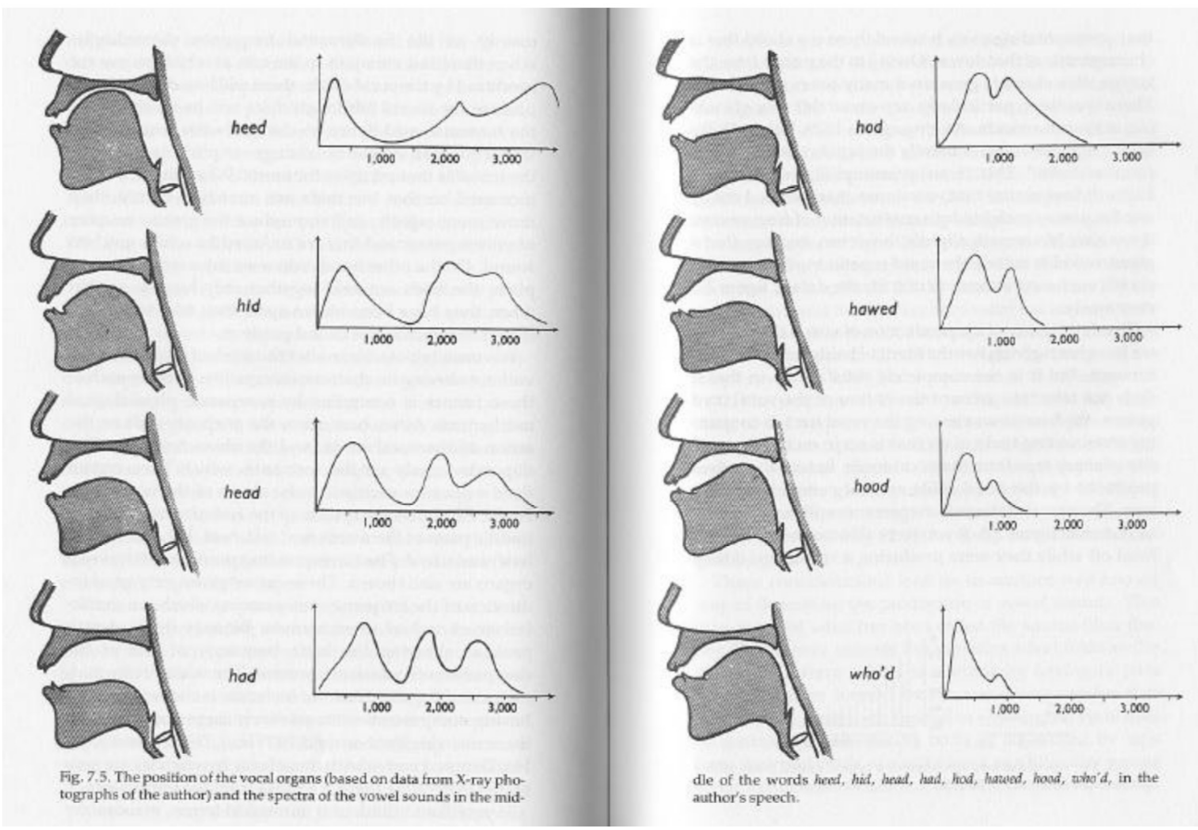
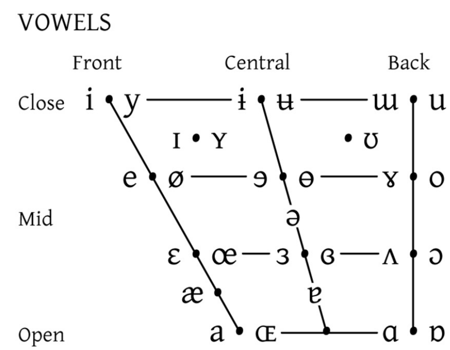


Fig. 7.5. The position of the vocal organs (based on data from X-ray photographs of the author) and the spectra of the vowel sounds in the middle of the words *heed, hid, head, had, hod, hawed, hood, who'd,* in the author's speech.



(Click on chart to get to IPA chart with audio.)